

A model of quantum vacuum as the origin of the speed of light

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Abstract. We show that the vacuum permeability μ_0 and permittivity ϵ_0 may originate from the magnetization and the polarization of continuously appearing and disappearing fermion pairs. We then show that if we simply model the propagation of the photon in vacuum as a series of transient captures within these ephemeral pairs, we can derive a finite photon velocity. Requiring that this velocity is equal to the speed of light constraints our model of vacuum. Within this approach, the propagation of a photon is a statistical process at scales much larger than the Planck scale. Therefore we expect its time of flight to fluctuate. We propose an experimental test of this prediction.

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1 Introduction

The vacuum permeability μ_0 , the vacuum permittivity ϵ_0 , and the speed of light in vacuum c are widely considered as being fundamental constants and their values, escaping any physical explanation, are commonly assumed to be invariant in space and time. In this paper, we propose a mechanism based upon a "natural" quantum vacuum description which leads to sensible estimations of these three electromagnetic constants. A consequence of this description is that μ_0 , ϵ_0 and c are not fundamental constants but observable parameters of the quantum vacuum: they can vary if the vacuum properties vary in space or in time. A similar analysis of the quantum vacuum, as the physical origin of the electromagnetism constants, has been proposed independently by Leuchs, Villar and Sanchez-Soto [1]. Although the two mechanisms are different, the original idea is the same: the physical electromagnetic constants emerge naturally from the quantum theory.

The paper is organized as follows. First we describe our model of the quantum vacuum filled with continuously appearing and disappearing fermion pairs. We show how μ_0 and ϵ_0 originate respectively from the magnetization and the electric polarization of these pairs. We then derive the photon velocity in vacuum by modeling its propagation as a series of interactions with the pairs. Finally, we predict statistical fluctuations of the transit time of photons across a fixed vacuum path.

2 An effective description of the quantum vacuum

The vacuum is assumed to be filled with continuously appearing and disappearing charged fermion pairs (ephemeral

particle-antiparticle pairs). We consider neither intermediate bosons nor supersymmetric particles. All known species of charged fermions are taken into account: the three families of charged leptons e , μ and τ and the three families of quarks (u, d) , (c, s) and (t, b) , including their three color states. This gives a total of 21 pair species, noted i .

An ephemeral fermion pair is assumed to be the product of the fusion of two virtual photons of the vacuum. Thus its total electric charge and total color are null. We suppose also that the spins of the two fermions of a pair are antiparallel, and that they are on their mass shell. The only quantity which is not conserved is therefore the energy and this is the reason for the limited lifetime of the pairs. We assume that first order properties can be deduced assuming that pairs are created with an average energy, not taking into account a full probability density of the pairs kinetic energy. Likewise, we will neglect the total momentum of the pair.

The average energy W_i of a pair is taken proportional to its rest mass energy $2m_i c^2$:

$$W_i = K_W 2m_i c^2 \quad (1)$$

where K_W is a constant, assumed to be independent from the fermion type. We take K_W as a free parameter, greater than unity. The value of K_W could be calculated if we knew the energy spectrum of the virtual photons together with their probability to create fermion pairs.

The pairs lifetime τ_i is given by the Heisenberg uncertainty principle ($W_i \tau_i = \hbar/2$). So

$$\tau_i = \frac{1}{K_W} \frac{\hbar}{4m_i c^2} \quad (2)$$

We assume that the ephemeral fermion pairs densities N_i are driven by the Pauli Exclusion Principle. Two pairs

containing two identical fermions in the same spin state cannot show up at the same time at the same place. However at a given location we may find 21 charged fermion pairs since different fermions can superpose spatially. In solid state physics the successful determination of Fermi energies[2] implies that one electron spin state occupies a hyper volume h^3 . We assume that concerning the Pauli principle, the ephemeral fermions are similar to the real ones. Noting Δx_i the spacing between identical i -type fermions and p_i their average momentum, the one dimension hyper volume is $p_i \Delta x_i$ and dividing by h should give the number of states which we take as one per spin degree of freedom. The relation between p_i and Δx_i reads $p_i \Delta x_i / h = 1$, or $\Delta x_i = 2\pi\hbar/p_i$.

We can express Δx_i as a function of W_i if we suppose the relativity to hold for the ephemeral pairs

$$\Delta x_i = \frac{2\pi\hbar c}{\sqrt{(W_i/2)^2 - (m_i c^2)^2}} = \frac{\lambda_{C_i}}{\sqrt{K_W^2 - 1}} \quad (3)$$

where $\lambda_{C_i} = h/(m_i c)$ is the Compton length associated to fermion i .

The pair density is defined as:

$$N_i \approx \frac{1}{\Delta x_i^3} = \left(\frac{\sqrt{K_W^2 - 1}}{\lambda_{C_i}} \right)^3 \quad (4)$$

Each pair can be produced only in the two fermion-antifermion spin combinations up-down and down-up. We define N_i as the density of pairs for a given spin combination.

Finally, we use the notation $Q_i = q_i/e$, where q_i is the i -type fermion electric charge and e the modulus of the electron charge.

3 The vacuum permeability

When a torus of a material is energized through a winding carrying a current I , the resulting magnetic flux density B is expressed as:

$$B = \mu_0 n I + \mu_0 M. \quad (5)$$

where n is the number of turns per unit of length and nI is the magnetic intensity in A/m . M is the corresponding magnetization induced in the material and is the sum of the induced magnetic moments divided by the corresponding volume. In an experiment where the current I is kept a constant and where we lower the quantity of matter in the torus, B decreases. As we remove all matter, B gets to a non zero value: $B = \mu_0 n I$ showing experimentally that the vacuum is paramagnetic with a vacuum permeability $\mu_0 = 4\pi \cdot 10^{-7} N/A^2$.

We propose a physical mechanism to produce the vacuum permeability from the elementary magnetization of the charged fermion pairs under a magnetic stress. Each charged ephemeral fermion carries a magnetic moment proportional to the Bohr magneton

$$\mu_i = \frac{e Q_i \hbar}{2m_i} = \frac{e Q_i c \lambda_{C_i}}{4\pi}. \quad (6)$$

Since the total spin of the pair is zero, and since fermion and antifermion have opposite charges, each pair carries twice the magnetic moment of one fermion.

If no external magnetic field is present, the magnetic moments point randomly in any direction resulting in a null global average magnetic moment. In the presence of an external magnetic field B , the coupling energy of the i -type pair to this field is $-2\mu_i B \cos \theta$, where θ is the angle between the magnetic moment and the magnetic field B . The energy of the pair is modified by this term and the pair lifetime is therefore a function of the orientation of its magnetic moment with respect to the applied magnetic field:

$$\tau_i(\theta) = \frac{\hbar/2}{W_i - 2\mu_i B \cos \theta}. \quad (7)$$

The pairs having their magnetic moment aligned with the field last a bit longer than the anti-aligned pairs. The resulting average magnetic moment $\langle M_i \rangle$ of a pair is therefore different from zero. It is aligned with the applied field. Its value is obtained integrating over θ with a weight proportional to the pairs lifetime:

$$\langle M_i \rangle = \frac{\int_0^\pi 2\mu_i \cos \theta \tau_i(\theta) 2\pi \sin \theta d\theta}{\int_0^\pi \tau_i(\theta) 2\pi \sin \theta d\theta}. \quad (8)$$

To first order in B , one gets:

$$\langle M_i \rangle \simeq \frac{4\mu_i^2}{3W_i} B. \quad (9)$$

The magnetic moment per unit volume produced by the i -type fermions is $M_i = 2N_i \langle M_i \rangle$, since one takes into account the two spin states per cell. The contribution $\tilde{\mu}_{0,i}$ of the i -type fermions to the vacuum permeability is thus given by $B = \tilde{\mu}_{0,i} M_i$ or $1/\tilde{\mu}_{0,i} = M_i/B$. Each species of fermions increases the induced magnetization and therefore the magnetic moment. By summing over all pair species, one gets the estimation of the vacuum permeability:

$$\frac{1}{\tilde{\mu}_0} = \sum_i \frac{M_i}{B} = \sum_i \frac{8N_i \mu_i^2}{3W_i} \quad (10)$$

Using Eq. (1), (4) and (6) and summing over all pair types, one obtains

$$\tilde{\mu}_0 = \frac{K_W}{(K_W^2 - 1)^{3/2}} \frac{24\pi^3 \hbar}{c e^2 \sum_i Q_i^2} \quad (11)$$

The sum $\sum_i Q_i^2$ is taken over all pair types. Within a generation the absolute values of the electric charges are 1, 2/3 and 1/3 in units of the positron charge. Thus for one generation the sum writes $(1 + 3 \times (4/9 + 1/9))$. The factor 3 is the number of colours. Each generation contributes equally, hence for the three families of the standard model

$$\sum_i Q_i^2 = 8 \quad (12)$$

One obtains:

$$\tilde{\mu}_0 = \frac{K_W}{(K_W^2 - 1)^{3/2}} \frac{3\pi^3 \hbar}{c e^2} \quad (13)$$

The calculated vacuum permeability $\tilde{\mu}_0$ is equal to the observed value μ_0 when

$$\frac{K_W}{(K_W^2 - 1)^{3/2}} = \mu_0 \frac{c e^2}{3\pi^3 \hbar} = \frac{4}{3} \frac{\alpha}{\pi^2} \quad (14)$$

which is obtained for $K_W \approx 31.9$, greater than one as required. In this model, we have ignored the charged loop contribution.

4 The vacuum permittivity

Consider a parallel-plate capacitor with a gas inside. When the pressure of the gas decreases, the capacitance decreases too until there is no more molecules in between the plates. The strange thing is that the capacitance is not zero when we hit the vacuum. In fact the capacitance has a very sizeable value as if the vacuum were a usual material body. The dielectric constant of a medium is coming from the existence of opposite electric charges that can be separated under the influence of an applied electric field \mathbf{E} . Furthermore the opposite charges separation stays finite because they are bound in a molecule. These opposite translations result in opposite charges appearing on the dielectric surfaces in regard to the metallic plates. This leads to a decrease of the effective charge, which implies a decrease of the voltage across the dielectric slab and finally to an increase of the capacitance. In our model of the vacuum the ephemeral charged fermion pairs are the pairs of opposite charge and the separation stays finite because the electric field acts only during the lifetime of the pairs. In an absolute empty vacuum, the induced charges would be null because there would be no charges to be separated and the capacitance of a parallel-plate capacitor would go to zero when one removes all molecules from the gas.

We show here that our vacuum filled by ephemeral fermions causes its electric charges to be separated and to appear at the level of 5.10^7 electron charges per square meter under an electric stress $E = 1 \text{ V/m}$. The mechanism is similar to the one proposed for the permeability. However, we must assume here that every fermion-antifermion ephemeral pair of the i -type bears a mean electric dipole d_i given by:

$$d_i = Q_i e \delta_i \quad (15)$$

where δ_i is the average separation between the two fermions of the pair. We assume that it does not depend upon the fermion momentum and we use the reduced Compton wavelength of the fermion $\lambda_{C_i}/(2\pi)$ as this scale:

$$\delta_i \simeq \frac{\lambda_{C_i}}{2\pi} \quad (16)$$

If no external electric field is present, the dipoles point randomly in any direction and their resulting average field

is zero. In presence of an external electric field \mathbf{E} , the mean polarization of these ephemeral fermion pairs produce the observed vacuum permittivity ϵ_0 . This polarization shows up due to the dipole lifetime dependence on the electrostatic coupling energy of the dipole to the field. In a field homogeneous at the δ_i scale, this energy is $d_i E \cos \theta$ where θ is the angle between the ephemeral dipole and the electric field \mathbf{E} . The electric field modifies the pairs lifetimes according to their orientation:

$$\tau_i(\theta) = \frac{\hbar/2}{W_i - d_i E \cos \theta} \quad (17)$$

As in the magnetostatic case, pairs with a dipole moment aligned with the field last a bit longer than the others. This leads to a non zero average dipole $\langle D_i \rangle$, which is aligned with the electric field \mathbf{E} and given, to first order in E , by:

$$\langle D_i \rangle \simeq \frac{d_i^2}{3W_i} E \quad (18)$$

We estimate the permittivity $\tilde{\epsilon}_{0,i}$ due to i -type fermions using the relation $P_i = \tilde{\epsilon}_{0,i} E$, where the polarization P_i is equal to the dipole density $P_i = 2N_i \langle D_i \rangle$, since the two spin combinations contribute. Thus:

$$\tilde{\epsilon}_{0,i} = 2N_i \frac{\langle D_i \rangle}{E} = 2N_i e^2 \frac{Q_i^2 \delta_i^2}{3W_i} \quad (19)$$

Each species of fermion increases the induced polarization and therefore the vacuum permittivity. By summing over all pair species, one gets the general expression of the vacuum permittivity:

$$\tilde{\epsilon}_0 = \frac{2e^2}{3} \sum_i \frac{N_i Q_i^2 \delta_i^2}{W_i} \quad (20)$$

Expressing the model parameters from Eq. (1), (4), (12) and (16), one gets:

$$\tilde{\epsilon}_0 = \frac{(K_W^2 - 1)^{3/2}}{K_W} \frac{e^2}{3\pi^3 \hbar c} \quad (21)$$

If we now use the value K_W given in Eq. (14) obtained from the derivation of the permeability, one gets the right numerical value for the permittivity: $\tilde{\epsilon}_0 = 8.85 \cdot 10^{-12} \text{ F/m}$.

We notice that the permeability and the permittivity do not depend upon the masses of the fermions. The electric charges and the number of species are the only important parameters. This is in opposition to the common idea that the energy density of the vacuum is the dominant factor[3].

5 The propagation of a photon in vacuum

When a real photon propagates in vacuum, it interacts with and is temporarily captured by an ephemeral pair. As soon as the pair disappears, it releases the photon to its

initial energy and momentum state. The photon continues to propagate with an infinite velocity. Then the photon interacts again with another ephemeral pair and so on. The delay on the photon propagation produced by these successive interactions implies that the velocity of light is finite.

By defining σ_i as the cross-section for a real photon to interact and to be trapped by an ephemeral i -type pair of fermions, the mean free path of the photon between two successive such interactions is given by:

$$\Lambda_i = \frac{1}{\sigma_i N_i} \quad (22)$$

where N_i is the numerical density of virtual i -type pairs.

Travelling a distance L in vacuum leads on average to $N_{stop,i}$ interactions on the i -type pairs, given by:

$$N_{stop,i} = \frac{L}{\Lambda_i} = L \sigma_i N_i \quad (23)$$

The photon may encounter the pair any time between its appearance and disappearance. The life time of a pair being τ_i , the photon will be stopped for an average time $\tau_i/2$. Each type of fermion pair contributes in increasing the propagation time of the photon. So, the total mean time \bar{T} for a photon to cross a length L is:

$$\bar{T} = \sum_i N_{stop,i} \frac{\tau_i}{2} \quad (24)$$

Using Eq. (23), we obtain the photon velocity \tilde{c} as a function of three parameters of the vacuum model:

$$\tilde{c} = \frac{L}{\bar{T}} = \frac{1}{\sum_i \sigma_i N_i \tau_i / 2} \quad (25)$$

Using Eq. (2) and (4), we get the expression

$$\tilde{c} = \frac{K_W}{(K_W^2 - 1)^{3/2}} \frac{16\pi}{\sum_i (\sigma_i / \lambda_{C_i}^2)} c \quad (26)$$

We now have to define the expression of the cross section σ_i . We know that it should not depend on the photon energy, otherwise the vacuum would become a dispersive medium. Also the interaction of a real photon with a pair must not exchange energy or momentum with the vacuum (for instance, Compton scattering is not possible). We assume the cross-section to be proportional to the geometrical cross-section of the pair $\lambda_{C_i}^2$, and to the square of the electric charge Q_i^2 . The cross-section is thus expressed as:

$$\sigma_i = k_\sigma Q_i^2 \lambda_{C_i}^2 \quad (27)$$

where k_σ is a constant which does not depend on the type of fermions.

The calculated photon velocity becomes:

$$\tilde{c} = \frac{K_W}{(K_W^2 - 1)^{3/2}} \frac{16\pi}{k_\sigma \sum_i Q_i^2} c \quad (28)$$

Using Eq. (12) and (14), one finally get:

$$\tilde{c} = \frac{8\alpha}{3\pi k_\sigma} c \quad (29)$$

The calculated velocity \tilde{c} of a photon in vacuum is equal to the observed value c when

$$k_\sigma = \frac{8}{3\pi} \alpha \quad (30)$$

It corresponds to a cross-section of $4 \cdot 10^{-26} \text{ m}^2$ on an ephemeral electron-positron pair, of the same order than the geometric transversal area of the pair, whose size is given in Eq. (16).

We note that the photon velocity depends only on the electrical charge units Q_i of the ephemeral charged fermions present in vacuum. It depends neither upon their masses, nor upon the vacuum energy density. We also remark that the average speed of the photon in our medium being c , the photon propagates, on average, along the light cone. As such, the effective average speed of the photon is independent of the inertial frame as demanded by relativity. This mechanism relies on the notion of an absolute frame for the vacuum at rest. It satisfies special relativity in the Lorentz-Poincaré sense.

6 Transit time fluctuations

An important consequence of our model is that stochastic fluctuations of the propagation time of photons in vacuum are expected, due to the fluctuations of the number of interactions of the photon with the virtual pairs and to the capture time fluctuations. Quantum gravity theories, which include stochastic fluctuations of the metric of compactified dimensions, predict also fluctuations of the propagation time of photons[4]. However observable effects are expected to be too small to be experimentally testable. It has been also recently predicted that the non commutative geometry at the Planck scale should produce a spatially coherent space-time jitter[5]. We show here that our effective model of photon propagation predicts fluctuations at a higher scale, which makes it experimentally testable with femtosecond pulses.

The propagation time T of a photon which crosses a distance L of vacuum is:

$$T = \sum_{i=1}^{N_{stop,i}} \sum_{k=1}^{t_{i,k}} t_{i,k} \quad (31)$$

where $t_{i,k}$ is the duration of the k^{th} interaction on i -type pairs and $N_{stop,i}$ the number of such interactions. The variance of T , due to the statistical fluctuations of the number of interactions and the fluctuation of the capture time is given by:

$$\sigma_T^2 = \sum_i \left(\sigma_{N_{stop,i}}^2 \bar{t}_{stop,i}^2 + N_{stop,i} \sigma_{t,i}^2 \right) \quad (32)$$

where $\bar{\tau}_{stop,i} = \tau_i/2$ is the average stop time on a i -type pair, $\sigma_{t,i}^2 = \tau_i^2/12$ its variance, and $\sigma_{N_{stop,i}}^2 = N_{stop,i}$ the variance of the number of interactions. Hence:

$$\sigma_T^2 = \frac{1}{3} \sum_i N_{stop,i}^2 \tau_i^2 = \frac{L}{3} \sum_i \sigma_i N_i \tau_i^2 \quad (33)$$

Once reduced, the current term of the sum is proportional to λ_{C_e} . Therefore the fluctuations of the propagation time are dominated by virtual e^+e^- pairs. Neglecting the other fermion species, and using $\sigma_e N_e \tau_e/2 = 1/(8c)$, one gets

$$\sigma_T^2 = \frac{\tau_e L}{12c} = \frac{\lambda_{C_e} L}{96\pi K_W c^2} \quad (34)$$

So

$$\sigma_T = \sqrt{\frac{L}{c}} \sqrt{\frac{\lambda_{C_e}}{c}} \frac{1}{\sqrt{96\pi K_W}} \quad (35)$$

In our simple model where $K_W = 31.9$, the predicted fluctuation is:

$$\sigma_T \approx 5 \cdot 10^{-2} \text{ fs.m}^{-1/2} \quad (36)$$

A way to search for these fluctuations is to measure a possible time broadening of a light pulse travelling a distance L of vacuum. This may be done using observations of brief astrophysical events, or dedicated laboratory experiments.

The strongest direct constraint from astrophysical observations is obtained with the very bright GRB 090510, detected by the Fermi Gamma-ray Space Telescope[6], at MeV and GeV energy scale. It presents short spikes in the 8 keV–5 MeV energy range, with the narrowest widths of the order of 4 ms (rms). Observation of the optical after glow, a few days later by ground based spectroscopic telescopes gives a common redshift of $z = 0.9$. This corresponds to a distance, using standard cosmological parameters, of about $2 \cdot 10^{26}$ m. Assuming that the observed width is correlated to the emission properties, this sets a limit for transit time fluctuations σ_T of about $0.3 \text{ fs.m}^{-1/2}$. It is important to notice that there is no expected dispersion of the bursts in the interstellar medium at this energy scale. If we move six orders of magnitude down in distances we arrive to kpc and pulsars. Short microbursts contained in main pulses from the Crab pulsar have been recently observed at the Arecibo Observatory telescope at 5 GHz[7]. The frequency-dependent delay caused by dispersive propagation through the interstellar plasma is corrected using a coherent dispersion removal technique. The mean time width of these microbursts after dedispersion is about 1 μ s, much larger than the expected broadening caused by interstellar scattering. Assuming again that the observed width is correlated to the emission properties, this sets a limit for transit time fluctuations of about $0.2 \text{ fs.m}^{-1/2}$.

The very fact that the predicted statistical fluctuations should go like the square root of the distance implies the exciting idea that experiments on Earth do compete with astrophysical constraints since we expect fluctuations in the femtosecond range at the kilometer scale.

An experimental setup using femtosecond laser pulses sent to a 100 m long multi-pass vacuum cavity equipped with metallic mirrors could be able to detect this phenomenon. With appropriate mirrors with no dispersion on the reflections, a pulse with an initial time width of 9 fs (FWHM) would be broadened after 30 round trips in the cavity, to an output time width of ~ 13 fs (FWHM). An accurate autocorrelation measurement could detect this effect.

7 Conclusions

We describe the ground state of the unperturbed vacuum as containing a finite density of charged ephemeral fermions antifermions pairs. Within this framework, ϵ_0 and μ_0 originate simply from the electric polarization and from the magnetization of these pairs when the vacuum is stressed by an electrostatic or a magnetostatic field respectively. The finite speed of light is due to successive transient captures of the photon by these virtual particles. Our calculated values for ϵ_0 , μ_0 and c are equal to the measured values when the fermion pairs are produced with an average energy of about 30 times their rest mass. This model is self consistent and it proposes a quantum origin to the three electromagnetic constants. The propagation of a photon being a statistical process, we predict fluctuations of the speed of light. This could be within the grasp of modern experimental techniques and we plan to assemble such an experiment.

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